Eton College King’s Scholarship Examination 2015

MATHEMATICS B

(One and a half hours)

Answer as many questions as you can.
Each of the ten questions is worth 10 marks.
Show all your working.
Calculators are allowed.
You need not answer the questions in the order set,
but if you move onto a question out of order, you must start it on a separate piece of paper.

Remember to write your candidate number on every sheet of answer paper used.

ADDITIONAL MATERIALS: ***CALCULATOR (provided by candidate)***

Do not turn over until told to do so.

1. (a) (i) How many even numbers are there from 100 to 1000 *inclusive*?

(ii) I write down m consecutive odd numbers, the smallest of which is n. Find in terms of m and n, the value of the largest odd number I wrote down.

(b) y is a positive whole number such that $\frac{y}{11}$ lies between $\frac{81}{13}$ and $\frac{98}{15}$. Find all possible values of y.

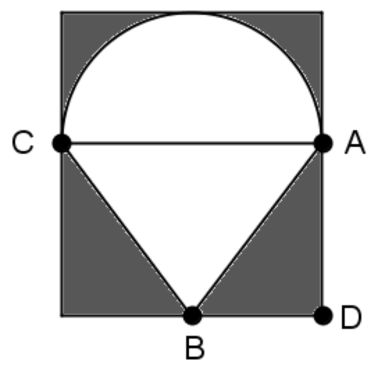
2. (a) I think of four numbers; the first is a half of the second number and also two-thirds of the third number and also three-quarters of the fourth number. Give that the numbers add to make 1260, find all four numbers.

(b) Claire has her birthday next month; it occurred to me that she will have her x^{th} birthday in the year x^2. Note: x is a whole number.

(i) Find x.

(ii) How old is Claire now?

3. A picture of an ice-cream in a cone is drawn using a semi-circle on top of an isosceles triangle ABC, with $\angle ACB = \angle CAB$. The picture fits exactly within a rectangle (with AC parallel to BD); the region outside the ice-cream and cone is shaded, as shown.



Given that $AD = 2.73\text{cm}$ and $AB = 3.05\text{cm}$, find, correct to 3 significant figures:

(a) the length AC;

(b) the shaded area.

4. (a) After removing 65% of the petrol from a tank, I have 10.5 litres left. How many litres did I remove?

(b) A certain area of my wall is covered in mould. I first remove mould from 10% of the area and then I remove mould in an area covering $120\ \text{cm}^2$. However, I leave it for a few days and the area covered by mould increases by 25% which returns it to its original size. By forming and solving an equation, find the original size.

5. *In this question, give any answers which are not whole numbers as mixed fractions.*

(a) Temperatures measured in degrees Celsius may be converted to degrees Fahrenheit by the following short calculation:

Multiply the temperature in Celsius by 9; divide the result by 5; add 32 to this result.

For example, 20° Celsius is equivalent to $20 \times 9 \div 5 + 32 = 68^\circ$ Fahrenheit.

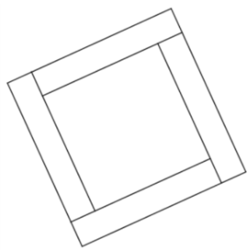
By forming and solving an equation, find the (chilly) temperature which has the same numerical value in Celsius as in Fahrenheit.

(b) King Julien devises his own temperature scale for his subjects. To convert from degrees Celsius to degrees Julien, the following procedure is followed:

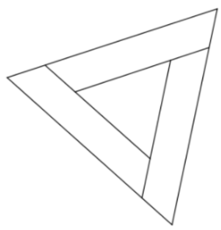
Add 7 to the temperature in degrees Celsius; divide the result by 10; multiply this result by 27.

(i) Show that 23 degrees Celsius is 81 degrees Julien.
(ii) Convert 100 degrees Julien to Celsius.
(iii) Find the temperature (in both Julien and Celsius) which has a numerical value in degrees Julien which is 75 higher than in degrees Celsius.

6. (a) As shown in the diagram, two squares lie within each other; the area in between is divided into four identical (that is, *congruent*) rectangles.
If the side length of the smaller square is q cm and the side length of the larger square is r cm, find the length of each of the sides of one of the rectangles in terms of q and r.



(b) As shown in the diagram, two equilateral triangles lie within each other; the area in between is divided into three congruent trapezia. If the side length of the smaller equilateral triangle is a cm and the side length of the larger equilateral triangle is b cm, find the length of each of the sides of one of the trapezia in terms of a and b.



7. The *Fibonacci* sequence starts as follows:

1, 1, 2, 3, 5, 8, 13, 21, 34, ...

Each term is created by adding together the two previous terms (1+1=2, 1+2=3, 2+3=5 etc.).

The *Lucas* sequence 1, 3, 4, 7, 11, 18, … is another sequence obeying the same rule, but starting with different numbers.

I have three more sequences in which each term is created by adding together the two previous terms. I only know a few terms of each sequence, and unknown terms are marked by a question mark.

(a) In sequence A, I only know the 1st and 3rd terms in the sequence:

11, ? , 14, ?

Find the two missing terms.

(b) In sequence B, I only know the 1st and 5th terms in the sequence:

15, ?, ?, ?, 51

By forming and solving an equation, find the missing three terms in the sequence.

(c) In sequence C, I only know that the 5th term is 37 and the 7th term is 96. Find the 9th term.

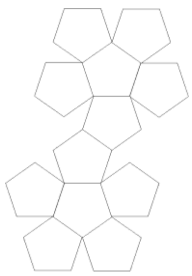
8. (a) A *space diagonal* of a cube (or indeed any polyhedron) is a line which joins two vertices which do not lie in the same face; by contrast, all other lines joining vertices are called *face diagonals*. Note: the sides of the cube are therefore face diagonals.

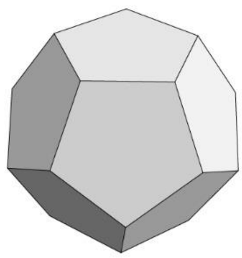
(i) Draw a diagram of a cube (either a net or a 3D view) to demonstrate that, from any single vertex, there are 6 face diagonals.

(ii) Find how many face diagonals a cube has in total.

(iii) Find how many space diagonals a cube has in total.

(b) A (regular) *dodecahedron* is a solid shape with twelve faces, all of which are identical regular pentagons; the diagrams show a net of the dodecahedron and a 3D view.





(i) Find how many vertices a dodecahedron has.

(ii) From each vertex, find how many face diagonals there are.

(iii) From each vertex, find how many space diagonals there are.

(iv) Find how many space diagonals a dodecahedron has in total.

9. In this question, we are restricting the variables x and y to be **positive whole numbers only.**

(a) ***Equation 1:*** $7x - 5y = 1$ x and y positive whole numbers only

(i) Show that ***Equation 1*** can be solved by setting $x = 3$ and state the corresponding value of y.

(ii) Show that $y = 11$ gives another solution pair.

(iii) Find two more solution pairs to ***Equation 1***.

(iv) By considering the pattern from all four of your solution pairs, suggest a method to find indefinitely more solutions to ***Equation 1***.

(b) ***Equation 2:*** $1651x - 1439y = 7$ x and y positive whole numbers only

Given that $257 \times 1439 = 224 \times 1651 - 1$

(i) find a positive whole number solution pair to ***Equation 2***;

(ii) find the *unique* solution pair to ***Equation 2*** which has the smallest value of x.

10. A 24-hour clock (pictured) is similar to a normal clock, but the hour hand rotates only once in a 24 hour period. The minute hand rotates as usual.

(a) Find the ratio of the angle rotated by the minute hand to the angle rotated by the hour hand, over the course of one minute. Give your ratio in whole numbers and reduced to its lowest form.

(b) Find the angle between the hour hand and the minute hand at the following times:

(i) 05:00 hours;

(ii) 00:30 hours;

(iii) 18:45 hours.

(c) Find the first times after midnight that the two hands are pointing:

(i) in opposite directions to each other;

(ii) in the same direction as each other.

Give your answers in the twenty-four hour clock to the nearest second.

[End of paper]